OCR
Oxford Cambridge and RSA

Wednesday 25 May 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4763/01 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4763/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 **(a)** In an investigation, small spheres are dropped into a long column of a viscous liquid and their terminal speeds measured. It is thought that the terminal speed V of a sphere depends on a product of powers of its radius r, its weight mg and the viscosity η of the liquid, and is given by

$$V = kr^{\alpha}(mg)^{\beta}\eta^{\gamma},$$

where k is a dimensionless constant.

(i) Given that the dimensions of viscosity are $\mathrm{ML^{-1}T^{-1}}$ find α, β and γ. **[6]**

A sphere of mass 0.03 grams and radius 0.2 cm has a terminal speed of $6\,\mathrm{m\,s^{-1}}$ when falling through a liquid with viscosity η. A second sphere of radius 0.25 cm falling through the same liquid has a terminal speed of $8\,\mathrm{m\,s^{-1}}$.

(ii) Find the mass of the second sphere. **[4]**

(b) A manufacturer is testing different types of light elastic ropes to be used in bungee jumping. You may assume that air resistance is negligible.

A bungee jumper of mass 80 kg is connected to a fixed point A by one of these elastic ropes. The natural length of this rope is 25 m and its modulus of elasticity is 1600 N. At one instant, the jumper is 30 m directly below A and he is moving vertically upwards at $15\,\mathrm{m\,s^{-1}}$. He comes to instantaneous rest at a point B, with the rope slack.

(i) Find the distance AB. **[5]**

The same bungee jumper now tests a second rope, also of natural length 25 m. He falls from rest at A. It is found that he first comes instantaneously to rest at a distance 54 m directly below A.

(ii) Find the modulus of elasticity of this second rope. **[4]**

2

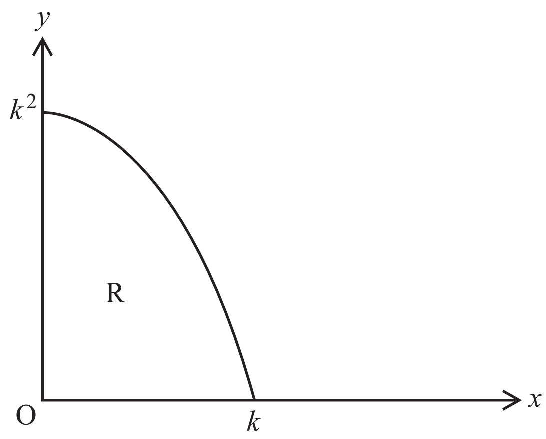

Fig. 2.1

The region R shown in Fig. 2.1 is bounded by the curve $y = k^2 - x^2$, for $0 \leqslant x \leqslant k$, and the coordinate axes. The x-coordinate of the centre of mass of a uniform lamina occupying the region R is 0.75.

(i) Show that $k = 2$. **[4]**

A uniform solid S is formed by rotating the region R through 2π radians about the x-axis.

(ii) Show that the centre of mass of S is at (0.625, 0). **[7]**

Fig. 2.2 shows a solid T made by attaching the solid S to the base of a uniform solid circular cone C. The cone C is made of the same material as S and has height 8 cm and base radius 4 cm.

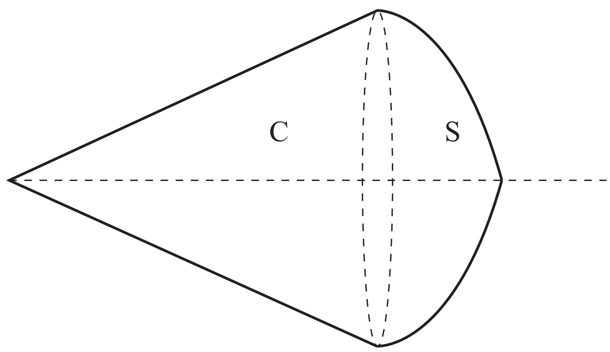

Fig. 2.2

(iii) Show that the centre of mass of T is at a distance of 6.75 cm from the vertex of the cone. [You may quote the standard results that the volume of a cone is $\frac{1}{3}\pi r^2 h$ and its centre of mass is $\frac{3}{4}h$ from its vertex.] **[4]**

(iv) The solid T is suspended from a point P on the circumference of the base of C. Find the acute angle between the axis of symmetry of T and the vertical. **[3]**

3

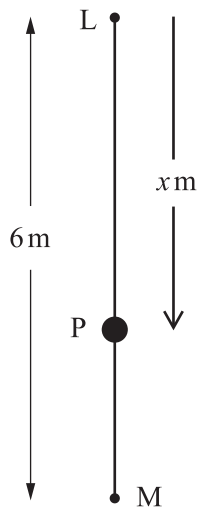

Fig. 3

One end of a light elastic string, of natural length 2.7 m and modulus of elasticity 54 N, is attached to a fixed point L. The other end of the string is attached to a particle P of mass 2.5 kg. One end of a second light elastic string, of natural length 1.7 m and modulus of elasticity 8.5 N, is attached to P. The other end of this second string is attached to a fixed point M, which is 6 m vertically below L. This situation is shown in Fig. 3.

The particle P is released from rest when it is 4.2 m below L. Both strings remain taut throughout the subsequent motion. At time t s after P is released from rest, its displacement below L is x m.

(i) Show that $\frac{d^2x}{dt^2} = -10(x-4)$. **[7]**

(ii) Write down the value of x when P is at the centre of its motion. **[1]**

(iii) Find the amplitude and the period of the oscillations. **[4]**

(iv) Find the velocity of P when $t = 1.2$. **[5]**

4 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. Particle P is projected so that it moves in complete vertical circles with centre O; there is no air resistance. A and B are two points on the circle, situated on opposite sides of the vertical through O. The lines OA and OB make angles α and β with the upward vertical as shown in Fig. 4.

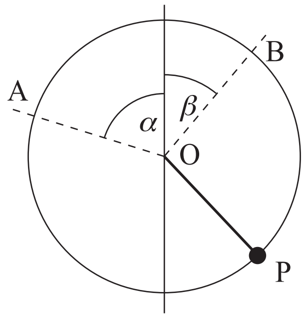

Fig. 4

The speed of P at A is $\sqrt{\frac{17ag}{3}}$. The speed of P at B is $\sqrt{5ag}$ and $\cos\beta = \frac{2}{3}$.

(i) Show that $\cos\alpha = \frac{1}{3}$. **[3]**

On one occasion, when P is at its lowest point and moving in a clockwise direction, it collides with a stationary particle Q. The two particles coalesce and the combined particle continues to move in the same vertical circle. When this combined particle reaches the point A, the string becomes slack.

(ii) Show that when the string becomes slack, the speed of the combined particle is $\sqrt{\frac{ag}{3}}$. **[2]**

The mass of the particle Q is km.

(iii) Find the value of k. **[9]**

(iv) Find, in terms of m and g, the instantaneous change in the tension in the string as a result of the collision. **[4]**

END OF QUESTION PAPER

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